NEWCASTLE GRAMMAR SCHOOL

Student Number: ____________________

2011

TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics

Examination Date: 17th August 2011

Examiner: K Armstrong

General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hours
- Write using a blue or black pen
- Write your student number on every booklet
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Each question attempted is to be returned in a separate Writing Booklet clearly marked Questions 1 etc.
- If required, additional booklets may be requested

Total marks - 120

- Attempt Questions 1-10
- All questions are of equal value

Question 1 (12 Marks) Use a SEPARATE writing booklet **Marks**

a) If $a = 0.45$, evaluate $\frac{1-a^2}{1+a^2}$ correct to 2 decimal places **2**

b) Simplify $\frac{4x-1}{2} - \frac{x+3}{3}$ **2**

c) Find a primitive of $2x^4 + e^{2x}$ **2**

d) Factorise completely $6x^3 - 48$ **2**

e) Solve for x : $|2x-1| \leq 5$ **2**

f) Write $\frac{2}{\sqrt{5}-1}$ with a rational denominator **2**

Question 2 (12 Marks) Use a SEPARATE writing booklet

a) Differentiate with respect to x

i) $x^2 e^{-x}$ **2**

ii) $\frac{x^2}{3x+1}$ **2**

b) Solve $\cos\theta = \frac{1}{\sqrt{2}}$ for $0 \leq \theta \leq 2\pi$ **2**

c) Find $\int \frac{2x^2}{2x^3 - 3}\, dx$ **2**

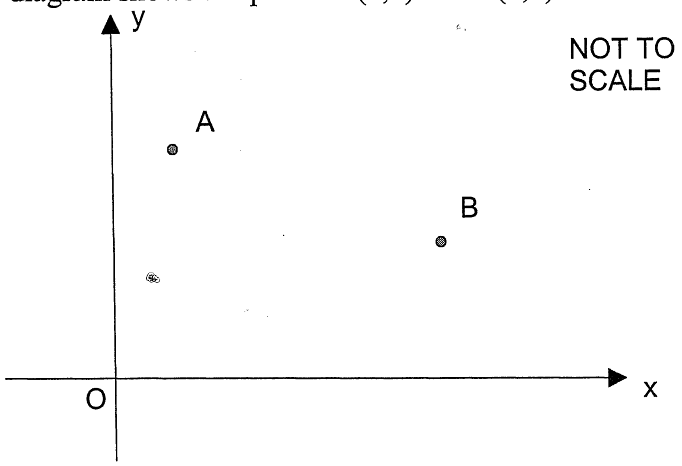
d) Evaluate $\int_0^{\frac{\pi}{3}} \sin 2x \, dx$ **2**

e) Find the equation of the normal to $y = \log_e x$ at the point (e, 1) **2**

Question 3 (12 Marks) Use a SEPARATE writing booklet **Marks**

a) Evaluate $\sum_{n=2}^{20} 6n-2$ **2**

b) The diagram shows the points A(1,3) and B(4,2)



i) Calculate the length of the interval AB. **2**

ii) Find the gradient of AB. **1**

iii) Find the equation of the line through A perpendicular to AB. **2**

iv) Show that the line in part **(iii)** passes through O. **1**

v) Calculate the length of AO and hence find the size of $\angle ABO$. **2**

vi) Find the coordinates of point C so that ABCO is a square. **2**

Question 4 (12 Marks) Use a SEPARATE writing booklet

a) A game is played in which two coloured dice are thrown once. The six faces of the red die are numbered 2, 4, 6, 8, 10 and 12. The six faces of the black die are numbered 1, 3, 5, 7, 9 and 11. The player wins if the number on the red die is larger than the number on the black die.

i) By drawing up a table of outcomes, or otherwise, calculate the probability of a player winning a game. **2**

ii) Calculate the probability that the player wins at least once in two successive games. **2**

b) If $\frac{d^2y}{dx^2} = 2 - \frac{4}{x^2}$ and at (1, 6) $\frac{dy}{dx} = 3$, find y in terms of x **3**

c) Find the value of $\log_{16} 128$ **1**

d) The sides of a triangle are 8cm, 10cm and 12cm

i) Calculate the size of the smallest angle (Answer to the nearest minute) **2**

ii) Calculate the area of the triangle (Answer correct to 2 decimal places) **2**

Question 5 (12 Marks) Use a SEPARATE writing booklet **Marks**

a) If α and β are roots of a quadratic equation $3x^2-4x-1=0$ find

i) $\alpha+\beta$ **1**

ii) $\alpha\beta$ **1**

iii) $\alpha^{-1}+\beta^{-1}$ **2**

iv) $\alpha\beta^3+\beta\alpha^3$ **3**

b) i) Sketch the curve $y=\tan\frac{x}{2}$ for $-2\pi\le x\le 2\pi$ **2**

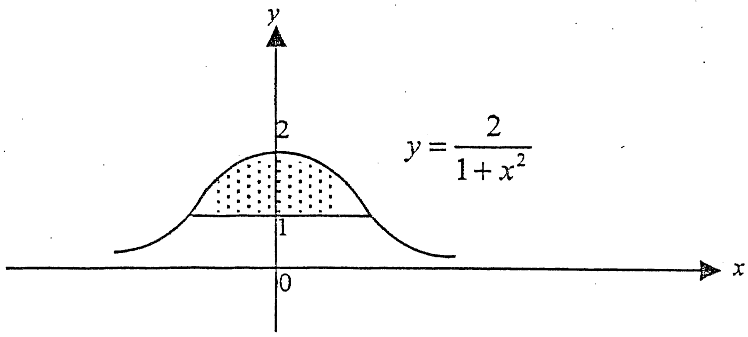
ii) State the period of $y=\tan\frac{x}{2}$ **1**

iii) Solve the equation $\tan\frac{x}{2}=1$ for $-2\pi\le x\le 2\pi$ **2**

Question 6 (12 Marks) Use a SEPARATE writing booklet

a) State the domain and range for the curve $y=\dfrac{4}{x-3}$ **2**

b)



Consider the region bounded by the curve $y=\dfrac{2}{1+x^2}$ and the line $y=1$ as shown in the diagram.

The region is revolved about the y-axis. Find the volume of the solid of revolution generated **4**

Question 6 continues next page

Question 6(continued) **Marks**

c) A chord AB of a circle with centre O has length 16cm. If the radius of the circle is 10cm, calculate

i) the size of angle AOB (Answer to the nearest degree) **2**

ii) the length of the minor arc AB(Answer correct to 1 decimal place) **2**

iii) the area of the minor segment formed by the chord AB(Answer to the nearest cm^2) **2**

Question 7 (12 Marks) Use a SEPARATE writing booklet

a) A particle moves in a straight line. At time t seconds, its displacement x metres from the origin is given by

$$x = 8t - 2t^2$$

i) Sketch the graph of x as a function of t, showing the vertex **1**

ii) Find the distance the particle travels in the first three seconds **2**

b) Find the coordinates of the point P on the curve $y = 3e^x - 2x$ at which the tangent to the curve is parallel to the line $y = 2x - 1$ **3**

c) For the curve $y = 2x^3 - 6x^2$

i) Determine all the stationary points and their nature. **2**

ii) Find what values of x the curve is concave down and decreasing. **2**

iii) Sketch the curve showing **all** main features in the domain $-2 \le x \le 4$ **2**

Question 8 (12 Marks) Use a SEPARATE writing booklet **Marks**

a) The population P of a growing town satisfies the equation

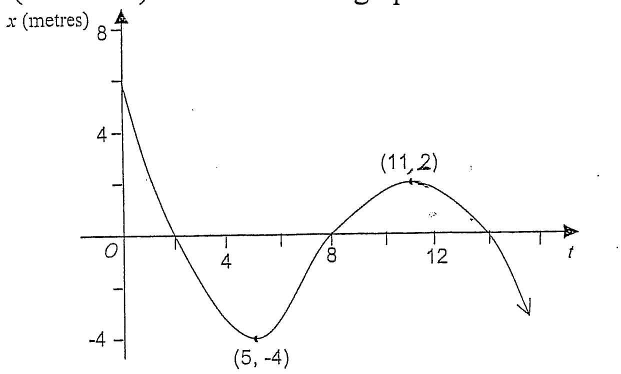
$$P = P_0 e^{kt} \qquad \text{where } t \text{ is time in years}$$

The initial population is 22 000 and five years later the population is 27 000

i) Find P_0 and k **3**

ii) When does the population reach 35 000? Give your answer correct to three significant figures. **2**

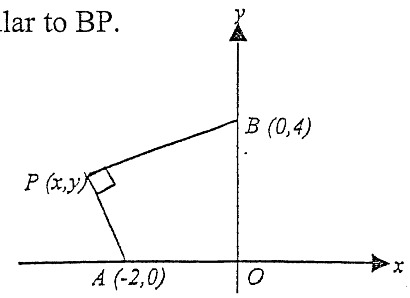
b) The displacement, x metres, from the origin of a particle moving in a straight line At any time (t seconds) is shown in the graph.



i) When was the particle at the origin? **1**

ii) How far did the particle travel in the first 11 seconds?(Show working) **2**

c) A(-2,0) and B(0,4) are two points in the number plane. P(x,y) is any point such that AP is perpendicular to BP.

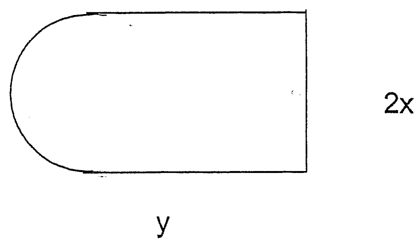


i) Prove that the equation of the locus of P(x,y) is $x(x+2)+y(y+4)=0$ **2**

ii) Deduce that the equation in (i) represents a circle and find its centre and radius. **2**

Question 9 (12 Marks) Use a SEPARATE writing booklet **Marks**

a) An artist's work is in the shape of a rectangle and a semi-circle as shown. The perimeter is to be 4 metres.



i) Show that $y=2-x-\dfrac{\pi x}{2}$ **2**

ii) Show that the area of the work can be expressed as $A=4x-2x^2-\dfrac{\pi x^2}{2}$ **2**

iii) Find the maximum area of the artwork. **3**

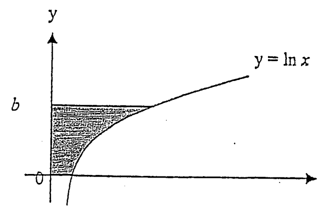
b) i) Show that $\sin^2 x\cos x=\cos x-\cos^3 x$ **1**

ii) Hence show that $\dfrac{d}{dx}\left(\sin x-\dfrac{1}{3}\sin^3 x\right)=\cos^3 x$ **2**

iii) Hence find $\int 3\cos^3 x\,dx$ **2**

Question 10 (12 Marks) Use a SEPARATE writing booklet **Marks**

a) If the shaded area is one square unit, find the value of b as an irrational number. **3**



b) Solve $e^{6x} - 7e^{3x} + 6 = 0$ **3**

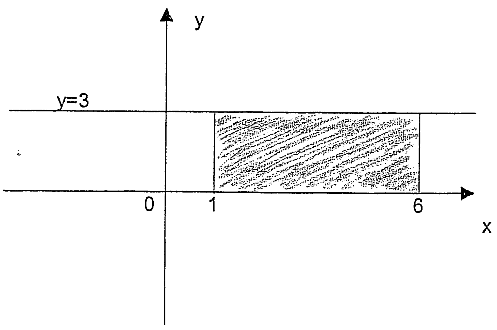
c) When Suzie was born, her grandparents set up a trust fund for her to receive on her 18th birthday. The conditions agreed upon for this trust fund involved investing \$100 every four months. Interest is calculated on the balance at the end of each year and is credited at the rate of 8%p.a.

i) Show that after two years the trust fund will amount to **2**

$$\$(300 \times 1.08^2 + 300 \times 1.08)$$

ii) How much will Suzie receive on her 18 th birthday from this trust fund? **2**

d) **2**



State two methods to find the volume generated by rotating the area shown in the diagram about the x-axis.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x>0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a>0, \quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x>0$

1a) $\frac{1-0.45^2}{1+0.45^2} = \frac{0.7975}{1.2025}$

$= 0.663201663$ — 1 mark

(2) → 0.66 (2DP) — 1 mark

b) $\frac{4x-1}{2} - \frac{x+3}{3}$

$= \frac{3(4x-1) - 2(x+3)}{6}$

$= \frac{12x-3-2x-6}{6}$ — ½ — 1 mark

$= \frac{10x-9}{6}$ — ½ — 1 mark

c) $\int 2x^4 + e^{2x}\, dx$

$= \frac{2x^5}{5} + \frac{1}{2}e^{2x}$ (no c necessary) (2) — 1 each part

d) $6x^3 - 48 = 6(x^3-8)$ — ½ — ½

½ off | mistake $= 6(x-2)(x^2+2x+4)$ — 1½ — 1½

e) $2x-1 \leq 5$ or $-2x+1 \leq 5$

$2x \leq 6$ $-2x \leq 4$

$x \leq 3$ or $x \geq -2$ — ½

Solution: $-2 \leq x \leq 3$ — 1

$-5 \leq 2x-1 \leq 5$
$-4 \leq 2x \leq 6$
$-2 \leq x \leq 3$
1 each

f) $\frac{2}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{2\sqrt{5}+2}{5-1}$ — 1 — 1 mark

$= \frac{2(\sqrt{5}+1)}{4}$ — ½

$= \frac{\sqrt{5}+1}{2}$ — ½ — 1 mark

2a) i) $\frac{d}{dx}\, x^2e^{-x} = x^2 \times -e^{-x} + e^{-x} \times 2x$ — 1 mark

$= xe^{-x}(-x+2)$ or $xe^{-x}(2-x)$ — 1 mark

ii) $\frac{d}{dx}\, \frac{x^2}{3x+1} = \frac{(3x+1)\times 2x - x^2 \times 3}{(3x+1)^2}$ — 1½ mark

$= \frac{6x^2+2x-3x^2}{(3x+1)^2}$ $\left(\frac{3x^2+2x}{(3x+1)^2}\right)$ or — ½ mark

$= \frac{x(3x+2)}{(3x+1)^2}$

b) cos is positive in 1st + 4th quadrants — ½ mark

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 45°$

$= \frac{\pi}{4}$ — ½ mark

$\therefore \theta = \frac{\pi}{4}$ or $\frac{7\pi}{4}$ — ½ mark ea

c) $\int \frac{2x^2}{2x^3-3}\, dx$

$\frac{1}{3}\ln(2x^3-3) + C$ — 1½ for integ, ½ for C

d) $\int_0^{\frac{\pi}{3}} \sin 2x$

$= \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{3}}$

$= -\frac{1}{2}\cos\frac{2\pi}{3} - \left(-\frac{1}{2}\cos 0\right)$ — 1 mark

$= \frac{1}{4} + \frac{1}{2}$

$= \frac{3}{4}$ — 1 mark

e) $y = \log_e x$

$y' = \frac{1}{x}$

when $x = e$

$m = \frac{1}{e}$ perp $m = -e$

$\therefore m_1m_2 = -1$ for perp. lines

$\therefore m$ of normal is $-e$

$y - 1 = -e(x-e)$

$y - 1 = -ex + e^2$

$y = -ex + 1 + e^2$

correct gradient — equation 1

3 $\sum_{n=2}^{20} 6n-2$ 10, 16, 22, ...

$a=10$, $d=6$, $n=19$, $S_n=?$

$S_n = \frac{n}{2}[2a+(n-1)d]$ — ½ formula

$= \frac{19}{2}[20+18\times6]$ — 1 correct subst.

$= 1216$ — ½ answer — (2)

b) i) $d=\sqrt{(4-1)^2+(2-3)^2}$ — 1 mark

$=\sqrt{3^2+(-1)^2}$

$=\sqrt{10}$ units $\doteqdot 3.16$ — 1 mark - answer — (2)

ii) m of AB $= \frac{2-3}{4-1}$

$= \frac{-1}{3}$ — 1 mark — (1)

iii) ⊥ grad is 3 (as $m_1m_2=-1$) — ½ for grad

$y-3 = 3(x-1)$ — 1½ for equation

$y-3 = 3x-3$

$3x-y=0$ — (2)

iv) LHS $= 3x-y$ RHS $=0$

$= 0-0$

$= 0$ — 1

∴ Line passes through O — (1)

v) $d=\sqrt{(3-0)^2+(1-0)^2}$ — 1 mark

$=\sqrt{10}$

As AO = AB + ∠BAO = 90° then ∠ABO = 45° — 1 mark — (2)

vi) Equation OC: $y=-\frac{x}{3}$

Equation BC $y-2=3(x-4)$

$y=3x-10$ — (2)

Solve simultaneously:

$-\frac{x}{3} = 3x-10$

$-x = 9x-30$

$30 = 10x$

$x=3$ $y=-1$

or

O is 3 down + 1 back from A then find C as 3 down + 1 back from B

i.e C is (3, -1) — 2 marks

Q4

a) i)

	2	4	6	8	10	12	Red
1	(12)	(14)	(16)	(18)	(1 10)	(1 12)	
3	32	(34)	(36)	(38)	(3 10)	(3 12)	
5	52	54	(56)	(58)	(5 10)	(5 12)	
7	72	74	76	(78)	(7 10)	(7 12)	
9	92	94	96	98	(9 11)	(9 12)	
11	11 2	11 4	11 6	11 8	11 10	(11 12)	
Black							

P(Winning) $= \frac{21}{36}$ $= \frac{7}{12}$ — 1 for table, 1 for answer

ii) P(At least 1) $= 1 - P(\text{none})$ — 1 working

$= 1 - \left(\frac{5}{12}\times\frac{5}{12}\right)$

$= 1 - \frac{25}{144}$

$= \frac{119}{144}$ — 1 answer

b) $y'' = 2-4x^{-2}$

$y' = 2x + 4x^{-1} + c$ when $x=1$ $y'=3$ — ½ for y'

$3 = 2+4+c$ — ½ for c

$c=-3$

$= 2x + \frac{4}{x} - 3$

$\therefore y = x^2 + 4\ln x - 3x + c$ — 1 for y

(1,6)

$6 = 1 + 4\ln 1 - 3 + c$ — $\frac{1}{2}$ for c

$c = 8$

$\therefore y = x^2 + 4\ln x - 3x + 8$ — $\frac{1}{2}$ for y

c) $\log_{16} 128 = x$

$\therefore 16^x = 128$ — 1

$(2^4)^x = 2^7$

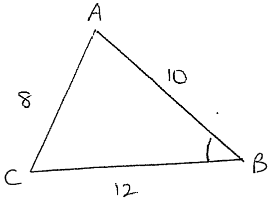
$4x = 7$

$x = \frac{7}{4}$

or $\log_{16} 128 = \frac{\log 128}{\log 16} = \frac{7}{4}$

d)

i)



Smallest angle is opposite the smallest side

$\cos A\hat{B}C = \frac{12^2 + 10^2 - 8^2}{2 \times 12 \times 10}$ — 1 correct formula & subst.

$\cos A\hat{B}C = 0.75$

$A\hat{B}C = 41° \; 25'$ — 1 answer

ii) $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 12 \times 10 \times \sin 41° 25'$ — 1

$= 39.69 \text{ cm}^2$ — 1

Q5 — (12)

a) $3x^2 - 4x - 1 = 0$

i) $\alpha + \beta = -\frac{b}{a}$

$= -\frac{-4}{3}$

$= \frac{4}{3}$ — (1) — 1 — 1

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{-1}{3}$ — (1) — 1 — 1

iii) $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\beta + \alpha}{\alpha\beta}$ — 1 — 1 algebra

$= \frac{4}{3} \div -\frac{1}{3}$ — (2) — 1

$= -4$ — 1 answer

iv) $\alpha\beta^3 + \beta\alpha^3 = \alpha\beta(\beta^2 + \alpha^2)$ — 1 — $\frac{1}{2}$ factorising

$= \alpha\beta\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$ — 1 — 1 algebra

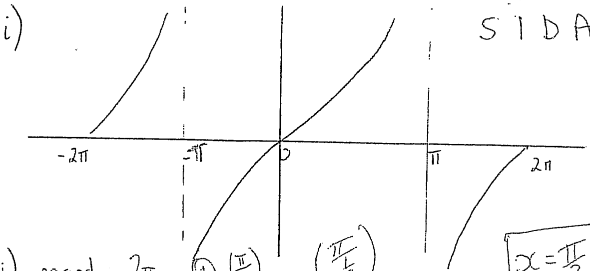
$= -\frac{1}{3} \times \left[\left(\frac{4}{3}\right)^2 - 2 \times -\frac{1}{3}\right]$ — 1 working

$= -\frac{1}{3} \times 2\frac{4}{9} = -\frac{1}{3} \times \frac{22}{9}$ — 1 subst.

$= -\frac{22}{27}$ — (3) — 1 — $\frac{1}{2}$ mark answer

b) i)

SIDA



(2) — 2 marks

ii) period $= 2\pi$ — (1) $(\frac{\pi}{6})$ $(\frac{\pi}{2})$

$x = \frac{\pi}{2}, -\frac{3\pi}{2}$ — (2) — $\frac{\pi}{2}, \frac{5\pi}{2}$ (1)

iii) $\therefore \frac{x}{2} = \frac{\pi}{4}$ or $-\frac{3\pi}{4}$ for $-\pi \le \frac{x}{2} \le \pi$

$x = \frac{\pi}{2}$ $x = \frac{3\pi}{2}$ — 1 each

6 a) $y = \frac{4}{x-3}$

Domain: all real x except $x = 3$ — 1 each

Range: all real x except $y = 0$. — 1 each.

b) $y = \frac{2}{1+x^2}$ make x^2 the subject

$y(1+x^2) = 2$

$y + yx^2 = 2$

$yx^2 = 2 - y$

$x^2 = \frac{2-y}{y}$

$x^2 = \frac{2}{y} - 1$ — 1

$\therefore V = \pi \int_a^b x^2 \, dy$

$= \pi \int_1^2 \frac{2}{y} - 1 \, dy$ — 1

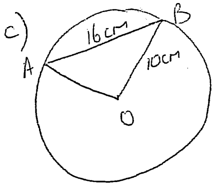
$= \pi \left[2\log y - y \right]_1^2$

$= \pi \left[2\log 2 - 2 - (2\log 1 - 1) \right]$ — 1

$= \pi \left[2\log 2 - 2 + 1 \right]$

$= \pi \left[2\log 2 - 1 \right]$ units^3 — 1

c)



i) $\cos \angle AOB = \frac{10^2 + 10^2 - 16^2}{2 \times 10 \times 10}$ — 1 formula + substitution

$\cos \angle AOB = -0.28$

$\angle AOB = 106^\circ$ — 1 answer

ii) $l = r\theta$ — ½ formula

$= 10 \times \frac{106 \times \pi}{180}$ — 1 substitution

$= 18.5$ (to 1 dec pl) cm — ½ answer

iii) $A = \frac{1}{2} r^2 (\theta - \sin\theta)$ — ½ formula

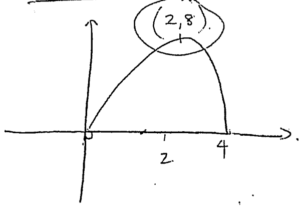
$= \frac{1}{2} \times 10^2 \times \left(\frac{106 \times \pi}{180} - \sin 106 \right)$ — 1 substitution

$= 44 \text{ cm}^2$ — ½ answer

7a) i) $x = 8t - 2t^2$

$2t(4 - t) = 0$

$t = 0$ or $t = 4$



½ vertex

½ x-intercept

(1)

ii) when $t = 3$

$x = 8 \times 3 - 2 \times 3^2$

$= 24 - 18$

$= 6$

∴ Distance is $= 10$ m (8+2 m)

1 for 6m

1 for 10m

(2)

b) $y = 3e^x - 2x$ // to $y = 2x - 1$

$y' = 3e^x - 2$ ½ $m = 2$ ½

∴ $2 = 3e^x - 2$

$4 = 3e^x$

$e^x = \frac{4}{3}$ ½

$\ln e^x = \ln \frac{4}{3}$

$x \ln e = \ln \frac{4}{3}$

$x = \ln \frac{4}{3}$

$y = 3e^{\ln \frac{4}{3}} - 2(\ln \frac{4}{3})$

$= 3 \times \frac{4}{3} - 2 \ln \frac{4}{3}$

$y = \frac{4}{3}$ 1

∴ P is $(\ln \frac{4}{3}, \frac{4}{3})$ ½

$(\ln \frac{4}{3},\ 4 - 2\ln \frac{4}{3})$

(0.287, 3.4)

No y 1½

(3)

½ for y'

½ for $m = 2$

½ for $e^x = \frac{4}{3}$

1 for y

½ for point

c) i). $y = 2x^3 - 6x^2$

$y' = 6x^2 - 12x$

$y'' = 12x - 12$

Stat. pts. occur when $y' = 0$

$6x^2 - 12x = 0$

$6x(x - 2) = 0$

$x = 0$ or $x = 2$

$y = 0$ or $y = -8$

when $x = 0$ $y'' < 0$ (-12)

∴ $(0,0)$ is maximum

when $x = 2$ $y'' > 0$ (12)

∴ $(2, -8)$ is minimum

(2)

1 for stat pts

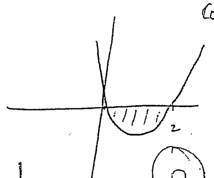
1 for nature

ii) decreasing $y' < 0$

$6x^2 - 12x < 0$

$6x(x-2) < 0$

∴ $0 < x < 2$. 1



(2)

Concave Down : $y'' < 0$

$12x - 12 < 0$

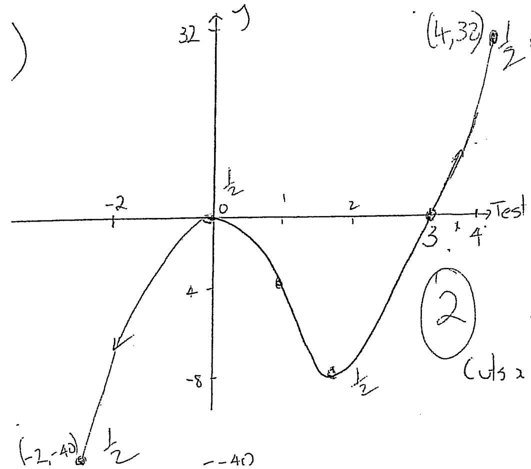
½ $x < 1$

$0 < x < 1$ ½

1 conc down

1 dec

iii)



(2)

½ Pt of inflexion occurs when $y'' = 0$

$12x = 12$

$x = 1$

$y = -4$

Not asked for

x	1⁻	1	1⁺
y''	-12	0	12

∴ $(1, -4)$ is a pt of inflexion

Cuts x axis when $y = 0$

$2x^2(x - 3) = 0$

$x = 0$ or $x = 3$.

8 a) $P = P_0 e^{kt}$

i) $P_0 = 22000$, $t = 5$, $P = 27000$

$27000 = 22000 e^{5k}$

$\frac{27}{22} = e^{5k}$

$\ln \frac{27}{22} = \ln e^{5k}$

$\ln \frac{27}{22} = 5k \ln e$

$k = \frac{1}{5} \ln \frac{27}{22}$

$\doteqdot 0.04095\ldots$

P_0 = 1 mark

k = 2 marks.

ii) $P = 35000$, $t = ?$

$35000 = 22000 e^{kt}$

$\frac{35}{22} = e^{kt}$

$\ln \frac{35}{22} = \ln e^{kt}$ 1 mark

$\ln \frac{35}{22} = kt \ln e$

$t = \frac{1}{k} \ln \frac{35}{22}$

$= 11 \cdot 3$ years (3SF) 1 mark

b) i) $x = 0$ at $t = 2, 8, 14$ 1 mark

ii)

Time	0-2	2-8	8-11
Distance	6	4+4	2

Total distance covered in the first 11 secs is $6 + 8 + 2 = 16$ m 2 marks

c) i) Since $AP \perp BP$ the product of their gradients is -1

$\therefore \frac{y}{x+2} \times \frac{y-4}{x} = -1$ 1 mark

or $x(x+2) + y(y-4) = 0$ 1 mark.

ii) $x^2 + 2x + y^2 - 4y = 0$

$x^2 + 2x + 1 + y^2 - 4y + 4 = 5$ 1 mark

$(x+1)^2 + (y-2)^2 = 5$

$\therefore$ Centre $(-1, 2)$ radius $\sqrt{5}$ 1 mark

Q9 i) Perimeter $= \frac{\pi \times 2x}{2} + y + 2x + y$ 1 mark

$4 = \pi x + 2y + 2x$

$\therefore 2y = -2x - \pi x + 4$

$y = 2 - x - \frac{\pi x}{2}$ (2) 1 mark.

ii) $A = 2xy + \frac{\pi x^2}{2}$ 1 mark

$= 2x\left(2 - x - \frac{\pi x}{2}\right) + \frac{\pi x^2}{2}$

$= 4x - 2x^2 - \pi x^2 + \frac{\pi x^2}{2}$

$= 4x - 2x^2 - \frac{\pi x^2}{2}$ (2). 1 Mark

iii) $\frac{dA}{dx} = 0$

$A = 4x - 2x^2 - \frac{\pi x^2}{2}$

$\frac{dA}{dx} = 4 - 4x - \pi x$ ½ 1 mark

$= 4 - (4 + \pi)x = 0$ ½

$(4 + \pi)x = 4$.

$x = \frac{4}{4 + \pi}$

9 $x = \frac{4}{4+\pi}$, $\frac{d^2A}{dx^2} = -4-\pi < 0$ $\therefore$ Maximum $\frac{1}{2}$ | 1 mark

$\therefore$ Maximum occurs when $x = \frac{4}{4+\pi}$ (0.56)

$A = 4\left(\frac{4}{4+\pi}\right) - 2\left(\frac{4}{4+\pi}\right)^2 - \frac{\pi}{2}\left(\frac{4}{4+\pi}\right)^2$

$= 1.12 m^2$ (~~to 2 dec pl~~) 1 (3) | 1 mark

b) i) $\sin^2 x \cos x = (1-\cos^2 x) \times \cos x$

$= \cos x - \cos^3 x$ (1) | 1 mark

ii) $\frac{d}{dx}\left(\sin x - \frac{1}{3}\sin^3 x\right) = \frac{d}{dx}\left(\sin x - \frac{1}{3}(\sin x)^3\right)$

$= \cos x - (\sin x)^2 \times \cos x$ 1 | 1 mark

$= \cos x - \cos x \sin^2 x$

from (i)

$= \cos x - (\cos x - \cos^3 x)$

$= \cos^3 x$ 1 (2) | 1 mark

iii) $\int 3\cos^3 x \, dx = 3\int \cos^3 x \, dx$ 1

$= 3\left[\sin x - \frac{1}{3}\sin^3 x\right] + c$ | 1 mark

$= 3\sin x - \sin^3 x + c$ 1 (2) | 1 mark

$\frac{1}{2}$ of

$\frac{16}{4\pi} - 2\frac{16}{(4+\pi)^2} - \frac{\pi}{2} \times \frac{16}{(4+\pi)^2}$

$= \frac{16(4+\pi) - 32 - 8\pi}{(4+\pi)^2}$

$= \frac{64 + 16\pi - 32 - 8\pi}{(4+\pi)^2}$

$= \frac{32 + 8\pi}{(4+\pi)^2} = \frac{8(\pi+4)}{(\pi+4)^2} = \frac{8}{\pi+4} \; m^2$

10 a) $y = \ln x$

$e^y = x$

$\int_0^b e^y \, dy = 1$ | 1 mark

$\left[e^y\right]_0^b = 1$

$e^b - e^0 = 1$ | 1 mark

$e^b - 1 = 1$

$e^b = 2$

$\therefore b = \ln 2$. | 1 mark

b) Let $m = e^{3x}$.

$\therefore m^2 - 7m + 6 = 0$ | 1 mark

$(m-6)(m-1) = 0$

$m = 6$ or $m = 1$

But $m = e^{3x}$

$\therefore e^{3x} = 6$ $\quad e^{3x} = 1$

$\ln e^{3x} = \ln 6$ $\quad \ln e^{3x} = \ln 1$

$3x \ln e = \ln 6$ $\quad 3x \ln e = 0$

$x = \frac{\ln 6}{3}$ $\quad 3x = 0$

$x = 0$. | 1 mark each

c) \$100 every 4 months = \$300 a year

i) First amount $A_1 = 300 \times 1.08^2$ | 1 setup

Second amount $A_2 = 300 \times 1.08^1$ | 1 mark

$\therefore$ After 2 years is $300 \times 1.08^2 + 300 \times 1.08$

ii) $A_{18} = \$300\left[1.08 + 1.08^2 + \ldots + 1.08^{18}\right]$ | 2 marks

$= \$300\left[\frac{1.08(1.08^{18} - 1)}{1.08 - 1}\right]$

$= \$12133.88$

d) $\pi\int_1^6 (3)^2\,dx$

$= \pi \int_1^6 9\,dx$

$= \pi \left[9x\right]_1^6$

$= \pi(54 - 9)$

$= 45\pi$ cubic units

or $V = \pi r^2 h$

$= \pi \times 3^2 \times 5$

$= 45\pi$ cubic units

1 mark each